Research Summary

Jiawei Sun

Electrical and Computer Engineering
University of Michigan

September, 2019

Outline

1. Differential Microphones Arrays based on Differential Equation
   - Linear DMA
   - Circular DMA
2. Distributed Algorithms of PCA
   - Backgrounding
   - Average Consensus Algorithm
   - Distributed PCA

Outline

1. Differential Microphones Arrays based on Differential Equation
   - Linear DMA
   - Circular DMA

2. Distributed Algorithms of PCA
   - Backgrounding
   - Average Consensus Algorithm
   - Distributed PCA

Linear DMA

Signal Model

- The basic mode of an uniform linear array (ULA) with M omnidirectional microphones is

$$\begin{aligned} y_m(k) &= x_m(k) + v_m(k) \\ &= x(k - t - \tau_m) + v_m(k), m = 1, 2, ..., M \end{aligned} \quad (1)$$

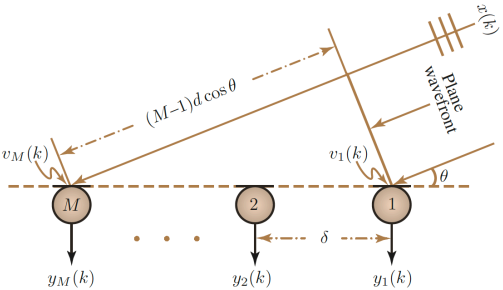

Figure: Uniform Linear Array

- where $x_m(k)$ is the source signal, t is the time which it takes form the signal to the first microphone, τ_m is the delay between the mth and the first microphones. [1]
- In the STFT domain,(1) can be expressed as

$$Y_m(\omega) = X(\omega)e^{-j(m-1)\omega\tau_0 \cos\theta} + V_m(\omega) \tag{2}$$

- In vectors form, we get

$$y(\omega) = [Y_1(\omega), Y_2(\omega), ..., Y_M(\omega)]^T \tag{3}$$
$$= d(\omega, \cos\theta)\mathrm{X}(\omega) + v(\omega) \tag{4}$$

where

$$d(\omega, \cos\theta) = \left[1, e^{-j\omega\tau_0 \cos\theta}, ..., e^{-j(M-1)\omega\tau_0 \cos\theta}\right]^T \tag{5}$$

is the phase-delay vector of length M.

- In order to recover the desired signal $X(\omega)$ from $y(\omega)$, a complex weight $H_m^*(\omega)$ is designed and applied to the output of each microphone. Mathematically, the beamformers output is

$$Z(\omega) = \sum_{m=1}^{M} H_m^*(\omega) Y_\mathrm{m}(\omega) = h^T(\omega) y(\omega) \tag{6}$$

- Mathematically, beampattern of a Nth-order DMA is written as

$$B[h(\omega), \theta] = d^H(\omega, \cos\theta)h(\omega) \tag{7}$$

$$= \sum_{m=1}^{M} H_m(\omega)e^{j(m-1)\omega\tau_0 \cos\theta} \tag{8}$$

Simplify (8) by McLaughlin expanion, we get

$$B_N(\theta) = \sum_{n=0}^{N} a_{N,n}\cos^n\theta \tag{9}$$

where $\mathrm{a}_{N,n}, n = 0, 1, ..., N$ are real coefficients.

- In the direction of the desired signal, i.e., $\theta = 0^\circ$, the directivity pattern must be equal to 1. Therefore, we should have

$$\sum_{n=0}^{N} a_{N,n} = 1 \tag{10}$$

Linear DMA

Linear equations solves beampattern coefficients

- The second-order directivity patterns have the form:

$$B_2(\theta) = (1 - a_{2,1} - a_{2,2}) + a_{2,1}\cos\theta + a_{2,2}\cos^2\theta \tag{11}$$

and they have 2 nulls at the angle θ_1 and θ_2, so we can write differential equation about $\mathrm{a}_{2,1}$ and $\mathrm{a}_{2,2}$,

$$\begin{cases} (1 - a_{2,1} - a_{2,2}) + a_{2,1}\cos\theta_1 + a_{2,2}\cos^2\theta_1 = 0 \\ (1 - a_{2,1} - a_{2,2}) + a_{2,1}\cos\theta_2 + a_{2,2}\cos^2\theta_2 = 0 \end{cases} \tag{12}$$

- By solving the equation, the most important shapes of patterns are as follows
  - Dipole: $a_{2,1} = 0$, $a_{2,2} = 1$, nulls at $\cos\theta = 0$.
  - Cardioid: $a_{2,1} = a_{2,2} = \frac{1}{2}$, nulls at $\cos\theta = 0$ and $\cos\theta = -1$.

Linear DMA

Linear equations solves beampattern coefficients

- As for Nth-order directivity patterns, they have N nulls at θ_1, θ_2,...,θ_N and the directivity pattern is equal to 1 in the direction of the desired signal. Based on the known conditions, we get

$$
\begin{cases}
\sum_{n=0}^{N} a_{N,n} = 1 \\
\sum_{n=0}^{N} a_{N,n} \cos^n\theta_1 = 0 \\
\sum_{n=0}^{N} a_{N,n} \cos^n\theta_2 = 0 \\
\vdots \\
\sum_{n=0}^{N} a_{N,n} \cos^n\theta_N = 0
\end{cases}
\tag{13}
$$

Solve the simultaneous linear equations and get $a_{N,0}$, $a_{N,2}$,...,$a_{N,N}$.

- By (9) and the multiple-angle formula

$$\cos^n\theta = \frac{1}{2^n}\sum_{k=0}^{n} C_n^k \cos(2k-n)\theta \tag{14}$$

we get,

$$B_N(\theta) = \sum_{n=0}^{N} b_{N,n}\cos n\theta \tag{15}$$

Linear DMA

Differential equations solves beampattern coefficients

- Take the first-order equations as example, it is written as

$$B_1(\theta)=b_{1,0} + b_{1,1}\cos\theta \tag{16}$$

As for a second-order constant coefficient differential equation,its corresponding characteristic equation is

$$y^{(2)} + n^2 y = 0 \tag{17}$$

$$r^2 + n^2 = 0 \tag{18}$$

so $r = \pm ni$, and its general solution is

$$y = C_1\cos(n\theta) + C_2\sin(n\theta) \tag{19}$$

Linear DMA

Differential equations solves beampattern coefficients

- Characteristic equation of a Nth-order constant coefficient differential equation corresponding to Nth-order DMA is

$$r(r^2+1^2)(r^2+2^2)\dots(r^2+N^2)=0 \tag{20}$$

To solve $(2N+1)$th-order differential equation needs 2N+1 initial conditions.

Linear DMA

Differential equations solves beampattern coefficients

- The directivity pattern must be equal to 1.

$$B_N(0^{\circ}) = 1 \tag{21}$$

Nth-order directivity patterns have N nulls at θ_1, θ_2,...,θ_N,

$$B_N(\theta_1) = 0, B_N(\theta_2) = 0, \dots, B_N(\theta_N) = 0 \tag{22}$$

Its first derivative only exists $\sin(n\theta)$ and so on we can get the rest N initial condition,

$$B_N^{(1)}(0) = 0, B_N^{(1)}(\pi) = 0, B_N^{(2)}(\frac{\pi}{2}) = 0, B_N^{(2)}(\frac{3\pi}{2}) = 0... \tag{23}$$

Outline

1. Differential Microphones Arrays based on Differential Equation
   - Linear DMA
   - Circular DMA
2. Distributed Algorithms of PCA
   - Backgrounding
   - Average Consensus Algorithm
   - Distributed PCA

- The beampattern of CDMA is defined as

$$B_N(\theta) = \sum_{n=0}^{N} a_{N,n} \cos^n(\theta - \theta_s) \tag{24}$$

where $\mathrm{a}_{N,n}, n = 0, 1, ..., N$ are real coefficients. In the direction of the desired signal, i.e., $\theta = \theta_s$, the directivity pattern must be equal to 1 [2].

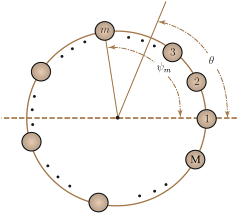

Figure: Circular DMA

Circular DMA

Differential Equations Solve Circular DMA

- By (24) and the multiple-angle formula

$$B_N(\theta - \theta_s) = \sum_{n=0}^{N} b_{N,n} \cos n(\theta - \theta_s) \tag{25}$$

$$= \sum_{n=0}^{N} b_{N,n} \cos n\theta_s \cos n\theta + b_{N,n} \sin n\theta_s \sin n\theta \tag{26}$$

Take the first-order equations as example, it is writtens as

$$B_1(\theta - \theta_s) = b_{1,0} + b_{1,1} \cos \theta_s \cos \theta + b_{1,1} \sin \theta_s \sin \theta \tag{27}$$

When $b_{1,0} = C$, $C_1 = b_{1,1} \cos \theta_s$, $C_2 = b_{1,1} \sin \theta_s$, the solution of this differential equation is equal to (27).

- To solve $(2N+1)$th-order differential equation needs 2N+1 initial conditions.

$$B_N(\theta_s) = 1 \tag{28}$$

Nth-order directivity patterns have N nulls at $\theta_1 - \theta_s$, $\theta_2 - \theta_s$,...,$\theta_N - \theta_s$,

$$B_N(\theta_1 - \theta_s) = 0, B_N(\theta_2 - \theta_s) = 0, \ldots, B_N(\theta_N - \theta_s) = 0 \tag{29}$$

Besides, we can get the rest N initial conditions,

$$\begin{aligned} &B_N^{(1)}(\theta_s) = 0, B_N^{(1)}(\theta_s + \pi) = 0, \\ &B_N^{(2)}(\theta_s + \frac{\pi}{2}) = 0, B_N^{(2)}(\theta_s + \frac{3\pi}{2}) = 0... \end{aligned} \tag{30}$$